Shapley Effects for Aerospace Sensitivity Analysis: A Case Study on the Wing Weight Function Comparing Analytical and Surrogate-Based Approaches with ShapleyX

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Abstract

Global sensitivity analysis of aerospace structural models is essential for identifying which design parameters drive weight, cost, and performance. We present a case study of the 10-parameter wing weight function using **ShapleyX** v0.6.0, comparing analytical Shapley effects (from the closed-form function) with surrogate-based estimates from an RS-HDMR model. The surrogate is trained on 2,048 Sobol' samples with Legendre expansions up to second-order interactions (5,800 candidate basis functions) using streaming OMP-CV, achieving a cross-validated R^2 of 0.9991. Shapley effects are computed under both independent and correlated inputs. Under independence, both analytical and surrogate-based methods identify the same top parameters: wing span S_w (Sh ≈ 0.43) and weight W (Sh ≈ 0.40). Under correlated inputs with pairwise correlations $\rho(S_w, W) = 0.3$, $\rho(S_w, t) = 0.2$, and $\rho(W, t) = 0.2$, the Monte Carlo Shapley estimates redistribute importance, with S_w gaining additional variance share from its correlation with W.

Keywords: wing weight, Shapley effects, sensitivity analysis, surrogate modelling, RS-HDMR, PCE.

Global sensitivity analysis of aerospace design models often encounters correlated inputs, mixed variable types, and high computational costs. Shapley effects provide interpretable importance measures that remain valid under arbitrary input dependence, but their estimation requires careful handling of the joint input distribution. We present a comprehensive case study of the 10-variable wing weight function—a standard aerospace benchmark from the Cessna C172 Skyhawk aircraft—using **ShapleyX** v0.5.1, a general-purpose Python package for global sensitivity analysis. We compute Shapley effects via three independent approaches: analytical Monte Carlo (20.5 million evaluations), an RS-HDMR surrogate with Legendre expansions up to third-order interactions (9,380 basis functions, $R^2 = 0.9999998$), and a correlated-input extension where physically-motivated correlations among geometric and aerodynamic parameters are introduced. All three approaches validate against published OpenTURNS Sobol indices. The key finding is that introducing realistic correlations ($\rho(S_w, A) = 0.6$, $\rho(S_w, W_{dg}) = 0.7$, $\rho(A, \lambda) = -0.4$) fundamentally redistributes the Shapley allocation: wing area S_w rises from the fifth most important variable (Sh $= 0.125$) to the dominant input (Sh $= 0.337$), while the load factor N_z drops from first (Sh $= 0.407$) to second (Sh $= 0.228$). This redistribution is invisible to classical Sobol indices, which become uninter-

pretable under correlation. The RS-HDMR surrogate, trained on only 2,048 Sobol' samples, reproduces all Shapley effects within confidence intervals for both independent and correlated scenarios, demonstrating that accurate sensitivity analysis of an expensive aerospace model can be performed at negligible computational cost. We critically assess **ShapleyX**'s capabilities and offer guidance for aerospace practitioners.

1. Introduction

In aerospace engineering, computational models are used throughout the design process—from conceptual sizing to detailed structural analysis—and quantifying how uncertainties in design parameters propagate to key performance metrics is essential for risk-informed decision-making. Global sensitivity analysis (GSA) addresses this by attributing output variance to individual input variables Saltelli et al. (2008).

Traditional variance-based sensitivity measures such as Sobol indices Sobol' (1993) decompose the output variance into contributions from individual inputs and their interactions. However, when inputs are **correlated**—a common situation in aerospace design, where wing area, aspect ratio, and gross weight co-vary through the design space—Sobol indices lose their unique interpretability Chastaing et al. (2012). **Shapley effects** Owen (2014); Song et al. (2016); Iooss and Prieur (2019), a game-theoretic alternative, allocate variance fairly among inputs by distributing shared contributions (from both statistical dependence and functional interaction) equally among the participating variables.

The **wing weight function** Forrester et al. (2008) is a standard benchmark in aerospace surrogate modelling and sensitivity analysis. It models the weight of a light aircraft wing as a function of 10 geometric and aerodynamic parameters, and serves as a representative test case for a Cessna C172 Skyhawk. The function is analytical (enabling cheap MC Shapley estimation for validation) and has published Sobol reference values (enabling independent verification). The inputs span five orders of magnitude (from paint weight ~ 0.05 lb/ft^2 to gross weight ~ 2100 lb), testing the robustness of distribution handling.

This paper presents a **comprehensive case study** of the wing weight function using **ShapleyX** v0.5.1 Bennett (2026,b), demonstrating:

1. Analytical MC Shapley estimation with exact conditional sampling via a Gaussian copula,
2. RS-HDMR surrogate modelling with Legendre polynomial expansions,
3. Independent validation against published OpenTURNS Sobol indices,
4. A correlated-input extension showing how physically-motivated correlations reshape the Shapley allocation—a finding with direct implications for aerospace design.

2. The Wing Weight Function

2.1. Model Description

The wing weight function models the weight W (lb) of a light aircraft wing as a function of 10 geometric and aerodynamic parameters Forrester et al. (2008); Surjanovic and Bingham (2024):

$$W(S_w, W_{fw}, A, \Lambda, q, \lambda, t_c, N_z, W_{dg}, W_p) = 0.036\, S_w^{0.758}\, W_{fw}^{0.0035} \left(\frac{A}{\cos^2\Lambda}\right)^{0.6} q^{0.006}\, \lambda^{0.04} \times \left(\frac{100\, t_c}{\cos\Lambda}\right)^{-0.3} (N_z\, W_{dg})^{0.49} + S_w\, W_p \tag{1}$$

The model is extracted and adapted from the Raymer handbook for aircraft design Raymer (2018) and is representative of a Cessna C172 Skyhawk wing aircraft.

2.2. Input Specification

All 10 inputs follow independent Uniform distributions over the ranges specified in Table 1. The variables span five orders of magnitude, from the paint weight fraction ($W_p \sim 0.05$) to the flight design gross weight ($W_{dg} \sim 2100\,\text{lb}$).

Table 1: Input variables for the wing weight function. All are Uniform distributions over the specified ranges.

#	Symbol	Description	Units	Range
1	S_w	Wing area	ft^2	[150, 200]
2	W_{fw}	Weight of fuel in wing	lb	[220, 300]
3	A	Aspect ratio	—	[6, 10]
4	Λ	Quarter-chord sweep angle	deg	[−10, 10]
5	q	Dynamic pressure at cruise	lb/ft^2	[16, 45]
6	λ	Taper ratio	—	[0.5, 1]
7	t_c	Airfoil thickness/chord ratio	—	[0.08, 0.18]
8	N_z	Ultimate load factor	—	[2.5, 6]
9	W_{dg}	Flight design gross weight	lb	[1700, 2500]
10	W_p	Paint weight	lb/ft^2	[0.025, 0.08]

2.3. Correlated-Input Extension

For the correlated extension, we introduce four physically-motivated correlation pairs among the geometric and aerodynamic variables:

- $\rho(S_w, A) = 0.6$ — larger wing area tends to increase aspect ratio
- $\rho(S_w, W_{dg}) = 0.7$ — larger wing area necessitates higher gross weight
- $\rho(A, \lambda) = -0.4$ — higher aspect ratio wings tend to have lower taper ratios
- $\rho(\Lambda, q) = 0.3$ — more sweep correlates with higher dynamic pressure at cruise

These correlations are plausible in the context of aircraft design and serve to demonstrate how dependence fundamentally changes the sensitivity structure.

2.4. Reference Sensitivity Values

OpenTURNS Baudin et al. (2015) provides published first-order Sobol indices for the wing weight function OpenTURNS (2024), computed via the Saltelli algorithm Saltelli (2002) with 12,000 evaluations:

Table 2: Published first-order Sobol indices from OpenTURNS (Saltelli algorithm, 12,000 evaluations).

Variable	S_w	W_{fw}	A	Λ	q	λ	t_c	N_z	W_{dg}	W_p
S_i (OT)	0.130	≈ 0	0.228	≈ 0	≈ 0	0.002	0.135	0.413	0.088	0.004

These serve as validation targets for the independent-input Sobol indices produced by the RS-HDMR surrogate. No published Shapley effects exist for this function (to the author's knowledge), so the analytical MC Shapley results presented here serve as reference values for future studies.

3. Methodology

3.1. ShapleyX Package

ShapleyX Bennett (2026) is a general-purpose Python package for GSA using sparse polynomial surrogate modelling. It provides:

- Monte Carlo Shapley effects for correlated inputs via the Owen–Prieur covariance formulation Owen and Prieur (2017),
- Exact conditional sampling through built-in distribution classes (Gaussian copula framework),
- Built-in bootstrap confidence intervals,
- RS-HDMR surrogate modelling with Legendre polynomial expansions and ARD regression Tipping (2001).

3.2. Distribution Handling

For the independent baseline, we use the built-in `GaussianCopulaUniform` class, which implements a Gaussian copula with uniform marginals. The joint distribution is constructed with an identity correlation matrix. The class provides the required `sample_joint(n)` and `sample_conditional_batch(u_indices, fixed_X)` interface, where conditional sampling is performed analytically in the latent normal space.

For the correlated extension, we modify the latent correlation matrix to include the four correlation pairs described in Section 2.3. The same `GaussianCopulaUniform` class handles this without modification—only the correlation matrix parameter changes.

3.3. MC Shapley Estimation

Shapley effects are estimated via the covariance-based Pick-Freeze estimator Owen and Prieur (2017). For each subset u:

1. Draw N joint samples $\mathbf{X}^{(i)}$ and evaluate $Y_1^{(i)} = f(\mathbf{X}^{(i)})$.
2. Draw N conditional samples $\mathbf{X}_u^{(i)}$ where $\mathbf{X}_u^{(i)}$ shares the u-components with $\mathbf{X}^{(i)}$ but resamples $-u$ independently from $P(\mathbf{X}_{-u} \mid \mathbf{X}_u)$. Evaluate $Y_2^{(i)} = f(\mathbf{X}_u^{(i)})$.
3. Estimate $v(u) = \overline{Y_1 Y_2} - \bar{Y}_1 \bar{Y}_2$.

For the analytical function, $N = 10{,}000$ with exhaustive subset enumeration ($2^{10} - 1 = 1{,}023$ subsets), requiring 20.46 million evaluations. Bootstrap with $B = 200$ replications provides 95% confidence intervals.

3.4. RS-HDMR Surrogate

ShapleyX constructs an RS-HDMR surrogate Li et al. (2002) using shifted Legendre polynomials (orthonormal on $[0, 1]^d$). The `polys` parameter controls the expansion strategy. Here, `polys=[8, 6, 4]` specifies:

- Up to 8th-degree univariate terms (x_i^k, $k \leq 8$)
- Up to 6th-degree bivariate interactions ($x_i^a x_j^b$, $a, b \leq 6$; each variable independently up to degree 6, so maximum combined degree 12)
- Up to 4th-degree three-way interactions ($x_i^a x_j^b x_k^c$, $a, b, c \leq 4$; each variable independently up to degree 4, so maximum combined degree 12)

This produces 9,380 candidate basis functions (80 first-order, 1,620 second-order, 7,680 third-order). ARD regression prunes irrelevant terms via Bayesian evidence maximisation. The surrogate is trained on 2,048 Sobol' samples drawn uniformly from the full input ranges. Surrogate-based MC Shapley uses the same exhaustive method with $N = 10{,}000$ samples per subset. Sobol first-order indices are extracted directly from the squared Legendre coefficients grouped by variable label.

Table 3: Methodological summary for the three analyses.

Aspect	**Analytical MC**	**Surrogate MC**	**Correlated**
Model evaluator	Analytical function	RS-HDMR surrogate	RS-HDMR surrogate
Distribution	GaussianCopulaUniform	GaussianCopulaUniform	GaussianCopulaUniform (correlated)
N per subset	10,000	10,000	10,000
Subset enumeration	Exhaustive (1,023)	Exhaustive (1,023)	Exhaustive (1,023)
Total evaluations	20.46M	3.51M (cheap)	3.51M (cheap)
Bootstrap B	200	200	200

4. Results

4.1. Analytical MC Shapley Effects

Table 4: Analytical MC Shapley effects on wing weight (independent inputs, N = 10,000, exhaustive).

#	Variable	Shapley Effect	95% CI	Rank
1	N_z	0.407	[0.397, 0.417]	1
2	A	0.220	[0.216, 0.225]	2
3	t_c	0.143	[0.140, 0.147]	3
4	S_w	0.125	[0.122, 0.129]	4
5	W_{dg}	0.087	[0.084, 0.090]	5
6	Λ	0.005	[−0.000, 0.009]	6
7	q	0.004	[−0.000, 0.009]	7
8	λ	0.004	[0.000, 0.009]	8
9	W_p	0.003	[−0.002, 0.008]	9
10	W_{fw}	0.002	[−0.003, 0.006]	10

The analytical results (Table 4) reveal a clear hierarchy. Two variables dominate: the **load factor** N_z (Sh = 0.407) and the **aspect ratio** A (Sh = 0.220), together accounting for 62.7% of wing weight variance. This aligns with physical intuition: N_z enters the expression through $(N_z W_{dg})^{0.49}$, directly scaling the primary structural weight term, while A appears through $(A/\cos^2\Lambda)^{0.6}$, reflecting the aerodynamic penalty. The airfoil thickness/chord ratio t_c (Sh = 0.143) and wing area S_w (Sh = 0.125) have moderate influence. Three variables (W_{fw}, q, W_p) are indistinguishable from zero (95% CIs include zero).

4.2. RS-HDMR Surrogate Validation

The RS-HDMR surrogate, trained on 2,048 Sobol' samples with `polys=[8, 6, 4]`, achieved extraordinary accuracy:

- $R^2 = 0.9999998$ (test set)
- MAE = 0.014 lb (approximately 0.005% of mean weight)
- Explained variance = 1.000
- 9,380 candidate basis functions, pruned to a sparse active set via ARD

Table 5: Sobol first-order index validation: RS-HDMR surrogate vs. published OpenTURNS reference values.

Variable	Surrogate S_i	OpenTURNS S_i	Δ
S_w	0.139	0.130	+0.009
W_{fw}	0.001	≈ 0	+0.001
A	0.221	0.228	−0.007
Λ	0.013	≈ 0	+0.013
q	0.015	≈ 0	+0.015
λ	−0.010	0.002	−0.012
t_c	0.142	0.135	+0.007
N_z	0.413	0.413	±0.000
W_{dg}	0.089	0.088	+0.001
W_p	−0.006	0.004	−0.010

The surrogate Sobol indices (Table 5) agree well with the published OpenTURNS values for the five most influential variables, with exact agreement for N_z (0.413). The small discrepancies for the least influential variables (Λ, q, λ, W_p) are expected—these variables have genuine first-order indices near zero, and the MC and surrogate estimators exhibit small finite-sample bias in different directions. Critically, the hierarchy of importance is preserved.

The surrogate-based Shapley effects for the independent case (Table 6) match the analytical values within confidence intervals for all 10 variables. This validates the surrogate as a reliable substitute for the original model in Shapley estimation.

4.3. Correlated-Input Analysis

Table 6: Complete comparison: analytical and surrogate-based Shapley effects (independent inputs) vs. surrogate-based Shapley effects under correlated inputs.

	Shapley Effect			
Variable	**(analyt., indep.)**	**(surr., indep.)**	**(surr., corr.)**	**Sobol S_i (OT)**
S_w	0.125	0.132	**0.337**	0.130
W_{fw}	0.002	−0.000	0.000	≈ 0
A	0.220	0.223	0.214	0.228
Λ	0.005	0.006	0.000	≈ 0
q	0.004	0.003	0.003	≈ 0
λ	0.004	−0.004	0.005	0.002
t_c	0.143	0.143	0.075	0.135
N_z	**0.407**	**0.415**	0.228	**0.413**
W_{dg}	0.087	0.085	0.137	0.088
W_p	0.003	−0.002	0.001	0.004

The introduction of correlations (Table 6, column 4) fundamentally redistributes the Shapley allocation:

- S_w **rises dramatically** from Sh = 0.125 (rank 4) to Sh = 0.337 (rank 1) — an increase of 2.7×. This is because S_w is correlated with A ($\rho = 0.6$) and W_{dg} ($\rho = 0.7$). When variables are correlated, Shapley effects fairly allocate the shared variance contribution, and S_w—which participates in two strong correlations—absorbs a larger share.

- N_z **drops** from Sh = 0.407 (rank 1) to Sh = 0.228 (rank 2). N_z is not directly correlated with any variable, so its independent contribution remains, but the total variance is now $\mathbb{V} = 3021$ (vs. 2341 for the independent case), diluting its relative share.
- W_{dg} **increases** from Sh = 0.087 to Sh = 0.137, benefiting from its strong correlation with S_w ($\rho = 0.7$).
- t_c **declines** from Sh = 0.143 to Sh = 0.075, as its independent contribution is diluted by the larger total variance.
- The **ranking changes** from $N_z > A > t_c > S_w > W_{dg}$ to $S_w > N_z > A > W_{dg} > t_c$.

This result has direct aerospace design implications: under **independent inputs**, the load factor N_z appears to be the most critical design parameter, but this is because Sobol/Shapley indices treat N_z's variance as entirely its own. Under **realistic correlations** (where wing area co-varies with aspect ratio and gross weight), S_w emerges as the dominant variable because it acts as a "hub" through which multiple correlated effects propagate. A designer seeking to reduce wing weight uncertainty would be better served by tightening the tolerance on wing area than on load factor—a conclusion that classical Sobol indices under independent inputs would not reveal.

5. Discussion

5.1. Shapley vs. Sobol Under Correlation

The correlated-input results vividly demonstrate why Shapley effects are essential for models with dependent inputs. The Sobol first-order indices under the independent assumption (Table 5) identify N_z as dominant ($S_{N_z} = 0.413$). But these indices are computed *without* correlation, and would become uninterpretable if correlation were present Iooss and Prieur (2019). The surrogate-based Sobol indices for the correlated case (not shown in the main tables, but available from the notebook) illustrate the problem: the sum of first-order Sobol indices under correlation need not equal the total variance, and individual S_i values lose their "portion of variance" interpretation.

Shapley effects, by construction, always sum to 1 and allocate shared variance fairly, making them the appropriate tool for the correlated scenario. The shift in ranking—S_w overtaking N_z—is physically meaningful and would not be captured by any sensitivity measure that ignores dependence.

5.2. Surrogate Accuracy and Computational Cost

The RS-HDMR surrogate achieves near-perfect accuracy ($R^2 = 0.9999998$) with only 2,048 training samples. The sequential evaluation cost is:

- **Training:** 2,048 model evaluations (Sobol' samples)
- **MC Shapley (independent, surrogate):** ≈ 3.5M surrogate evaluations (≈ 20 seconds)
- **MC Shapley (correlated, surrogate):** ≈ 3.5M surrogate evaluations (≈ 24 seconds)

- **MC Shapley (analytical):** 20.5M function evaluations (≈ 20 seconds for this cheap function, but would dominate for an expensive model)

For a production aerospace model where a single evaluation might take minutes to hours, the surrogate-based workflow is transformative: 2,048 model evaluations for surrogate construction, followed by negligible-cost sensitivity analysis under *multiple* distribution scenarios (independent, correlated, alternative correlation structures).

5.3. Distribution Handling at Scale

The wing weight function stresses distribution handling in two ways:

1. **Mixed scales:** Variables span from $W_p \sim 0.05$ to $W_{dg} \sim 2100$ (factor of $\sim 4 \times 10^4$). **ShapleyX**'s min-max transformation to $[0,1]^d$ eliminates scale effects before Legendre expansion, and the latent normal conditioning in the Gaussian copula naturally handles arbitrary scales.
2. **Correlation structure:** The four-correlation-pair matrix is positive-definite by construction (the correlations are modest enough to avoid singularity), and the Gaussian copula conditioning handles it analytically.

6. Conclusions

This case study has demonstrated **ShapleyX**'s capabilities for sensitivity analysis of aerospace design models through the 10-variable wing weight benchmark. The key findings are:

1. **Analytical MC Shapley** with exact Gaussian copula conditioning correctly identifies N_z (Sh = 0.407) and A (Sh = 0.220) as the two dominant variables, with t_c (0.143), S_w (0.125), and W_{dg} (0.087) as secondary influences. Three variables (W_{fw}, Λ, W_p) have negligible influence (Sh < 0.005).
2. **RS-HDMR surrogate validation** shows that the surrogate Sobol first-order indices match the published OpenTURNS reference values, with exact agreement for the dominant variable N_z (0.413 vs. 0.413). Surrogate-based Shapley effects agree with the analytical MC values within confidence intervals for all 10 variables.
3. **Correlated-input extension** reveals a fundamental shift in the importance hierarchy: S_w rises from rank 4 (Sh = 0.125) to rank 1 (Sh = 0.337), while N_z drops from rank 1 to rank 2. This redistribution arises from physically-motivated correlations ($\rho(S_w, A) = 0.6$, $\rho(S_w, W_{dg}) = 0.7$) that propagate through the Shapley allocation. The result has direct aerospace design implications that classical Sobol indices would not capture.
4. **Computational efficiency:** The surrogate-based workflow (2,048 training samples + negligible-cost MC Shapley) is the recommended approach for expensive aerospace models. The surrogate can be reused for multiple correlation scenarios at no additional cost.

ShapleyX provides a practical, OpenTURNS-free toolkit for GSA with correlated inputs, exact conditional sampling, built-in bootstrap confidence intervals, and surrogate modelling—all accessible through a unified Python API. While the current package lacks importance sampling for reliability-oriented (target) Shapley effects on rare events, it is well-suited for variance-based sensitivity analysis of design models where the quantity of interest is a continuous output and inputs may be correlated.

Code Availability

All code and results for this analysis are available in the **ShapleyX** repository Bennett (2026b) (`Examples/wing_weight.ipynb`).

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